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Fundamental Stage Design of Countercurrent Contact System: Solute Transfers between two Immiscible Solvents

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Abstract : *The Kremser's equations have been serving successfully for many years as limiting design and analytical criterion for idealized countercurrent absorption, desorption and extraction systems that contains immiscible solvents. The performance equations developed by the Kremser subsequently revisited by Mott Sounders and George Granger Brown, and elaborated the same, which is popularly known as Kremser - Sounders-Brown equations. Much later G. E. Goring critically analyzed the Kremser - Sounders- Brown equations gave better insight. All these equations relate absorption factor or stripping factor or extraction factor, inlet and outlet mole fraction of the streams to the number of stages. This work, we have presented the derivation for the Kremser - Sounders- Brown equation in more simplified way in terms of mole ratio co-ordinates for the case of absorption and desorption. Further, the physical meanings underlying these performance equations were presented explicitly.*

Keywords: *Absorption, Countercurrent stage Operation, Desorption, Immiscible solvents, Kremser-Sounder-Brown equation, Mole ratio co-ordinates*

I. Introduction

The amount of solute transferred between two immiscible solvent streams either gas-liquid or liquid-liquid in a countercurrent contact depends on the relative flow rates of the streams, the equilibrium distribution ($y_n = mx_n$) of the solute between solvent phases, and the number of contacting states provided. A compact analytical relationship among these variables was first developed by Kremser and subsequently it is modified by Mott Sounders and George Granger Brown and is critically analyzed later by G. E. Goring for ideal cases where the equilibrium distribution is assumed to linear. Solute free (i.e., mole ratios) coordinates are very useful in staged operation [1-6]. In stage analysis mole ratios are usually designated by Y and X, whereas mole fractions are designated by y and x for E stream and R stream respectively [4, 5]. Mole ratios are equal to mole fractions for very dilute systems. However, mole ratios are used in the following derivation.

II. Derivation

The Consider a cascade having N_p stages in which two distinctly different streams are brought in contact and allowed to attain equilibrium and also separated at equilibrium. In such situation solute transfer occurs in two ways [5]. Either transfer from $E \rightarrow R$ or $R \rightarrow E$. This is usually denoted as absorption or desorption for the case of transfer of solute from Gas$\rightarrow$Liquid or Liquid$\rightarrow$Gas respectively. Let E_s and R_s are the nontransferable component flow rates. Therefore, the relation between E and E_s is $E_s= E(1-y)$. Similarly, for R stream the relation between R and R_s is $R_s= R(1-x)$. In the following sections a compact analytical relationship among absorption factor/ stripping factor, relative flow rates of the streams, the equilibrium distribution ($Y_n= mX_n$) of the solute between solvent phases, and the number of contacting stages was developed for two cases.

Case-I: Countercurrent cascade: The transfer between E phase and R phase [5]

transfer from $E \rightarrow R$ (i.e., for better understanding the transfer of solute is from Gas$\rightarrow$Liquid)

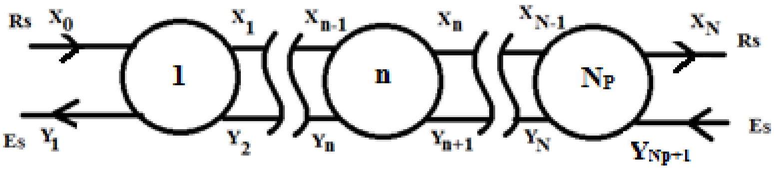

Figure 1. Countercurrent multistage cascade where transfer is from $E \rightarrow R$

n^{th} stage material balance

$$E_sY_{n+1} + R_sX_{n-1} = E_sY_n + R_sX_n \tag{1}$$

$$E_sY_{n+1} - E_sY_n = R_sX_n - R_sX_{n-1} = R_s\,{}^{Y_n}\!/_{m} - R_s\,{}^{Y_{n-1}}\!/_{m} \tag{2}$$

$$Y_{n+1} - Y_n = \frac{R_sY_n}{E_sm} - \frac{R_sY_{n-1}}{E_sm} \tag{3}$$

$$let\ {}^{R_s}\!/_{E_sm} = A$$

$$Y_n + AY_n = Y_{n+1} + AY_{n-1} \tag{4}$$

$$Y_n = \frac{Y_{n+1} + AY_{n-1}}{1+A} \tag{5}$$

For n =1

$$Y_1 = \frac{Y_2 + AY_0}{1+A} \tag{6}$$

For n =2

$$Y_2 = \frac{Y_3 + AY_1}{1+A} \tag{7}$$

Substituting For Y_1

$$Y_2 = \frac{Y_3 + A\left((Y_2 + AY_0) \big/ (1+A) \right)}{1+A} \tag{8}$$

$$Y_2(1+A)^2 - AY_2 = Y_3(1+A) + A^2 Y_0 \tag{9}$$

$$Y_2 = \frac{Y_3(A+1) + A^2 Y_0}{A^2 + A + 1} \tag{10}$$

For n =3

$$Y_3 = \frac{Y_4 + AY_2}{1+A} \tag{11}$$

Substituting Y_2

$$Y_3 = \frac{Y_4 + A\left(Y_3(A+1) + A^2 Y_0 \big/ A^2 + A + 1 \right)}{1+A} \tag{12}$$

$$Y_3(1+A)\left(A^2 + A + 1\right) - Y_3\left(A^2 + A\right) = Y_4\left(A^2 + A + 1\right) + A^3 Y_0 \tag{13}$$

$$Y_3 = \frac{Y_4\left(A^2 + A + 1\right) + A^3 Y_0}{A^3 + A^2 + A + 1} \tag{14}$$

Therefore for n = n

$$Y_n = \frac{Y_{n+1}\left(A^{n-1} + A^{n-2} + ...A + 1\right) + A^n Y_0}{A^n + A^{n-1} + ... + A + 1} \tag{15}$$

Multiply and divide with $(A-1)$ factor gives on RHS gives

$$Y_n = \frac{Y_{n+1}(A-1)\left(A^{n-1} + A^{n-2} + ...A + 1\right) + A^n (A-1) Y_0}{(A-1)\left(A^n + A^{n-1} + ... + A + 1\right)} \tag{16}$$

$$Y_n = \frac{Y_{n+1}\left(A^n - 1^n\right) + A^n (A-1) Y_0}{\left(A^{n+1} - 1^{n+1}\right)} = \frac{Y_{n+1}\left(A^n - 1\right) + A^n (A-1) Y_0}{\left(A^{n+1} - 1\right)} \tag{17}$$

$$Y_n = \frac{Y_{n+1}\left(A^n - 1\right) + \left(A^{n+1} - A^n\right) Y_0}{\left(A^{n+1} - 1\right)} \tag{18}$$

$$Y_n A^{n+1} - Y_n = Y_{n+1} A^n - Y_{n+1} + A^{n+1} Y_0 - A^n Y_0 \tag{19}$$

$$Y_n A^{n+1} - A^{n+1} Y_0 = Y_{n+1} A^n - A^n Y_0 + Y_n - Y_{n+1} \tag{20}$$

$$\left(Y_n - Y_0\right) A^{n+1} = \left(Y_{n+1} - Y_0\right) A^n + Y_n - Y_{n+1} \tag{21}$$

Add and subtract Y_0 on right side

$$\left(Y_n - Y_0\right) A^{n+1} = \left(Y_{n+1} - Y_0\right) A^n + Y_n - Y_{n+1} + Y_0 - Y_0 \tag{22}$$

$$\left(Y_n - Y_0\right) A^{n+1} = \left(Y_{n+1} - Y_0\right) A^n - \left(Y_{n+1} - Y_0\right) + \left(Y_n - Y_0\right) \tag{23}$$

$$\left(Y_n - Y_0\right) A^{n+1} - \left(Y_n - Y_0\right) = \left(Y_{n+1} - Y_0\right) A^n - \left(Y_{n+1} - Y_0\right) \tag{24}$$

$$\left(A^{n+1} - 1\right)\left(Y_n - Y_0\right) = \left(Y_{n+1} - Y_0\right)\left(A^n - 1\right) \tag{25}$$

$$\frac{Y_{n+1} - Y_0}{Y_n - Y_0} = \frac{A^{n+1} - 1}{A^n - 1} \tag{26}$$

Implies the above equation is true for $n = N_p$

$$\frac{Y_{N_p+1} - Y_0}{Y_{N_p} - Y_0} = \frac{\left(A^{N_p+1} - 1\right)}{\left(A^{N_p} - 1\right)} \tag{27}$$

From overall material balance

$$E_s Y_{N_p+1} + R_s X_0 = E_s Y_1 + R_s X_{N_p} \tag{28}$$

$$E_s Y_{N_p+1} + R_s X_0 - E_s Y_1 = R_s X_{N_p} = R_s \frac{Y_{N_p}}{m} \tag{29}$$

$$\frac{E_s m}{R_s} Y_{N_p+1} + m X_0 - \frac{E_s m}{R_s} Y_1 = Y_{N_p} \tag{30}$$

$$\frac{Y_{N_p+1} - Y_1}{A} + m X_0 = Y_{N_p} \quad \because Y_0 = m X_0 \tag{31}$$

$$\frac{Y_{N_p+1} - Y_1}{A} + Y_0 = Y_{N_p} \tag{32}$$

Combining following two equation gives

$$\frac{Y_{N_p+1}-Y_0}{Y_{N_p}-Y_0}=\frac{\left(A^{N_p+1}-1\right)}{\left(A^{N_p}-1\right)} \;\&\; \frac{Y_{N_p+1}-Y_1}{A}+Y_0=Y_{N_p} \tag{33}$$

$$\frac{Y_{N_p+1}-Y_0}{Y_{N_p+1}-Y_1}=\frac{\left(A^{N_p+1}-1\right)}{\left(A^{N_p+1}-A\right)} \tag{34}$$

On reversing Eq.34 we get

$$\frac{Y_{N_p+1}-Y_1}{Y_{N_p+1}-Y_0}=\frac{A^{N_p+1}-A}{A^{N_p+1}-1} \tag{35}$$

The physical meaning of the above expression is:

For $Y_0 = 0$ (i.e., fresh R-stream) the LHS is fraction absorbed

$$\frac{Y_{N_p+1}-Y_1}{Y_{N_p+1}}=\frac{A^{N_p+1}-A}{A^{N_p+1}-1} \tag{36}$$

$$\text{Fraction absorbed}=\frac{Y_{N_p+1}-Y_1}{Y_{N_p+1}}=\frac{\text{input composition} - \text{output composition}}{\text{input composition}} \tag{37}$$

$$\text{Fraction unabsorbed}=\frac{Y_1}{Y_{N_p+1}}=\frac{\text{output composition}}{\text{input composition}} \tag{38}$$

$$\frac{Y_1}{Y_{N_p+1}}=\frac{A-1}{A^{N_p+1}-1} \tag{39}$$

$$\frac{Y_0-Y_1}{Y_{N_p+1}-Y_0}=\frac{(1-A)}{\left(A^{N_p+1}-1\right)}=\frac{\frac{1}{A}-1}{A^{N_p}-1/A} \tag{40}$$

$$\frac{Y_1-Y_0}{Y_{N_p+1}-Y_0}=\frac{1-\frac{1}{A}}{A^{N_p}-\frac{1}{A}} \tag{41}$$

$$A^{N_p}-\frac{1}{A}=\frac{Y_{N_p+1}-Y_0}{Y_1-Y_0}\left(1-\frac{1}{A}\right) \tag{42}$$

$$A^{N_p}=\frac{Y_{N_p+1}-Y_0}{Y_1-Y_0}\left(1-\frac{1}{A}\right)+\frac{1}{A} \tag{43}$$

Applying logarithm on both sides gives

$$N_p = \frac{\log\left(\frac{Y_{N_p+1} - Y_0}{Y_1 - Y_0}\left(1 - \frac{1}{A}\right) + \frac{1}{A}\right)}{\log(A)} \tag{44}$$

$$N_p = \frac{\log\left(\frac{Y_{N_p+1} - mX_0}{Y_1 - mX_0}\left(1 - \frac{1}{A}\right) + \frac{1}{A}\right)}{\log(A)} \tag{45}$$

For Fresh R the concentration of solute in incoming R-stream is $X_0=0$

Therefore the number ideal plates for transfer of solute from E to Fresh R is as follows

$$N_p = \frac{\log\left(\frac{Y_{N_p+1}}{Y_1}\left(1 - \frac{1}{A}\right) + \frac{1}{A}\right)}{\log(A)} \tag{46}$$

Suppose the inlet for E stream is denoted as Y_1 (i.e. inlet) and out let is denoted as Y_2 as follows

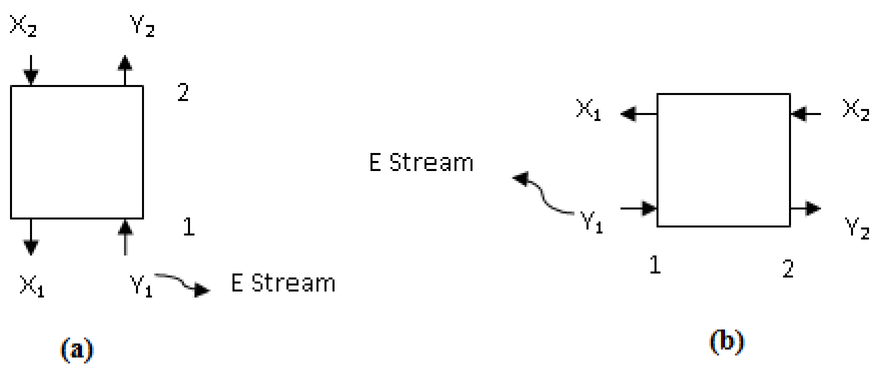

Figure 2 . (a) Vertical gas- liquid contactor where E steam at bottom; (b) Horizontal gas- liquid contactor where E-stream at left end

Therefore the number of plates N_p is

$$N_p = \frac{\log\left(\frac{Y_1}{Y_2}\left(1 - \frac{1}{A}\right) + \frac{1}{A}\right)}{\log(A)} \tag{47}$$

Case-II: Countercurrent cascade: The transfer between R phase and E phase [5] transfer from $R \rightarrow E$ (i.e., for better understanding the transfer of solute is from Liquid $\rightarrow$ Gas)

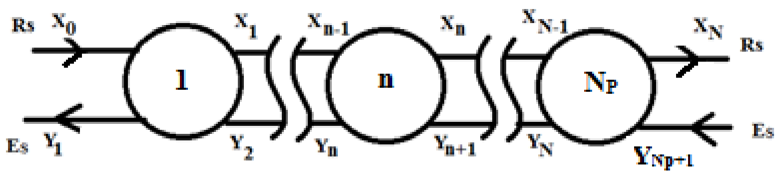

Figure 3. Countercurrent multistage cascade where transfer is from $R \rightarrow E$

n^{th} stage material balance

$$E_s Y_{n+1} + R_s X_{n-1} = E_s Y_n + R_s X_n \tag{48}$$

$$R_s X_{n-1} - R_s X_n = E_s Y_n - E_s Y_{n+1} = E_s m X_n - E_s m X_{n+1} \tag{49}$$

$$R_s X_{n-1} + E_s m X_{n+1} = \left(E_s m + R_s\right) X_n \tag{50}$$

let $S = \dfrac{E_s m}{R_s}$

$$X_n = \frac{R_s X_{n-1} + E_s m X_{n+1}}{E_s m + R_s} = \frac{X_{n-1} + S X_{n+1}}{S+1} \tag{51}$$

For n =1

$$X_1 = \frac{X_0 + S X_2}{S+1} \tag{52}$$

For n =2

$$X_2 = \frac{X_1 + S X_3}{S+1} \tag{53}$$

Combining Eq.52 and Eq.53 gives

$$X_2 = \left\{ \frac{\dfrac{X_0 + S X_2}{S+1} + S X_3}{S+1} \right\} \tag{54}$$

$$X_2 (S+1)^2 = X_0 + S X_2 + S(S+1) X_3 \tag{55}$$

$$X_2 \left(1 + S^2 + 2S\right) = X_0 + S X_2 + S X_3 + S^2 X_3 \tag{56}$$

$$X_2 \left(1 + S^2 + S\right) = X_0 + \left(S + S^2\right) X_3 \tag{57}$$

$$X_2 = \frac{X_0 + \left(S + S^2\right)X_3}{1 + S + S^2} \tag{58}$$

For $n = 3$

$$X_3 = \frac{X_2 + SX_4}{S + 1} \tag{59}$$

Combining Eq.58 and Eq.59

$$X_3 = \frac{\left[\dfrac{X_0 + \left(S + S^2\right)X_3}{\left(1 + S + S^2\right)}\right] + SX_4}{\left(1 + S\right)} \tag{60}$$

$$X_3(1 + S)\left(1 + S + S^2\right) = X_0 + \left(S + S^2\right)X_3 + \left(S + S^2 + S^3\right)X_4 \tag{61}$$

$$X_3(1 + S)\left(1 + S + S^2\right) = X_0 + S(1 + S)X_3 + \left(S + S^2 + S^3\right)X_4 \tag{62}$$

$$X_3(1 + S)\left(1 + S + S^2\right) - S(1 + S)X_3 = X_0 + \left(S + S^2 + S^3\right)X_4 \tag{63}$$

Taking $X_3(1 + S)$ common from LHS gives

$$X_3(1 + S)\left\{1 + S + S^2 - S\right\} = X_0 + \left(S + S^2 + S^3\right)X_4 \tag{64}$$

$$X_3\left(1 + S + S^2 + S^3\right) = X_0 + \left(S + S^2 + S^3\right)X_4 \tag{65}$$

$$X_3 = \frac{X_0 + \left(S + S^2 + S^3\right)X_4}{1 + S + S^2 + S^3} \tag{66}$$

For n = n

$$X_n = \frac{X_0 + \left(S + S^2 . + ... + S^n\right)X_{n+1}}{1 + S + S^2 + S^3 + ... + S^n} \tag{67}$$

$$X_n\left(1 + S + S^2 + ... + S^n\right) = X_0 + \left(S + S^2 + ... + S^n\right)X_{n+1} \tag{68}$$

Adding and subtracting X_{n+1} on RHS gives

$$X_n\left(1 + S + S^2 + ... + S^n\right) = X_0 + \left(S + S^2 + ... + S^n\right)X_{n+1} + X_{n+1} - X_{n+1} \tag{69}$$

$$X_n\left(1 + S + S^2 + ... + S^n\right) = X_0 + \left(1 + S + S^2 + ... + S^n\right)X_{n+1} - X_{n+1} \tag{70}$$

$$X_n\left(1+S+S^2+.....+S^n\right)-\left(1+S+S^2+........+S^n\right)X_{n+1}=X_0-X_{n+1} \tag{71}$$

Taking $\left(1+S+S^2+........+S^n\right)$ as common on LHS

$$\left(1+S+S^2+........+S^n\right)\left(X_n-X_{n+1}\right)=X_0-X_{n+1} \tag{72}$$

$$\frac{X_0-X_{n+1}}{X_n-X_{n+1}}=1+S+S^2+........+S^n=\frac{1-S^{n+1}}{1-S}(or)\frac{S^{n+1}-1}{S-1} \tag{73}$$

Therefore for $n = N_p$

$$\frac{X_0-X_{N_p+1}}{X_{N_p}-X_{N_p+1}}=1+S+S^2+........+S^{N_p}=\frac{1-S^{N_p+1}}{1-S}(or)\frac{S^{N_p+1}-1}{S-1} \tag{74}$$

$$\frac{X_0-X_{N_p+1}}{X_{N_p}-X_{N_p+1}}=\frac{S^{N_p+1}-1}{S-1}=\frac{S^{N_p}-\frac{1}{S}}{1-\frac{1}{S}} \tag{75}$$

$$\frac{X_0-X_{N_p+1}}{X_{N_p}-X_{N_p+1}}=\frac{S^{N_p}-\frac{1}{S}}{1-\frac{1}{S}} \tag{76}$$

$$\frac{X_0-X_{N_p+1}}{X_{N_p}-X_{N_p+1}}\left(1-\frac{1}{S}\right)=S^{N_p}-\frac{1}{S} \tag{77}$$

$$S^{N_p}=\frac{X_0-X_{N_p+1}}{X_{N_p}-X_{N_p+1}}\left(1-\frac{1}{S}\right)+\frac{1}{S} \tag{78}$$

Applying log on both sides gives

$$N_p\log(S)=\log\left(\frac{X_0-X_{N_p+1}}{X_{N_p}-X_{N_p+1}}\left(1-\frac{1}{S}\right)+\frac{1}{S}\right) \tag{79}$$

$$N_p=\frac{\log\left(\frac{X_0-X_{N_p+1}}{X_{N_p}-X_{N_p+1}}\left(1-\frac{1}{S}\right)+\frac{1}{S}\right)}{\log(S)} \tag{80}$$

$$N_p = \frac{\log\left(\frac{X_0 - \frac{Y_{N_p+1}}{m}}{X_{N_p} - \frac{Y_{N_p+1}}{m}}\left(1-\frac{1}{S}\right)+\frac{1}{S}\right)}{\log(S)} \tag{81}$$

(Or)

In terms of absorption factor

$$N_p = \frac{\log\left(\frac{X_0 - \frac{Y_{N_p+1}}{m}}{X_{N_p} - \frac{Y_{N_p+1}}{m}}(1-A)+A\right)}{\log\left(\frac{1}{A}\right)} \tag{82}$$

Equation 75 can be written in a meaningful way as follows

$$\frac{X_0 - X_{N_p+1}}{X_{N_p} - X_{N_p+1}} - 1 = \frac{S^{N_p+1}-1}{S-1} - 1 \tag{83}$$

$$\frac{X_0 - X_{N_p}}{X_{N_p} - X_{N_p+1}} = \frac{S^{N_p+1}-S}{S-1} \tag{84}$$

(Or)

$$\frac{X_0 - X_{N_p}}{X_{N_p} - \frac{Y_{N_p+1}}{m}} = \frac{S^{N_p+1}-S}{S-1} \tag{85}$$

(or)

$$\frac{X_0 - X_{N_p}}{X_0 - \frac{Y_{N_p+1}}{m}} = \frac{S^{N_p+1}-S}{S^{N_p+1}-1} \tag{86}$$

The physical meaning of the above expression is:

For pure E stream $Y_{N_p+1} = 0$

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$$\frac{X_0 - X_{N_p+1}}{X_0} = \frac{\text{input composition} - \text{output composition}}{\text{input composition}} = \textit{fraction desorbed}$$

$$\frac{X_0 - X_{N_p}}{X_0} = \frac{S^{N_p+1} - S}{S^{N_p+1} - 1} = \textit{fraction desorbed} \tag{87}$$

$$\frac{X_{N_p}}{X_0} = \frac{S - 1}{S^{N_p+1} - 1} = \textit{fraction undesorbed} \tag{88}$$

Suppose the inlet for R stream X_2 (i.e. inlet) and out let is denoted as X_1 as follows

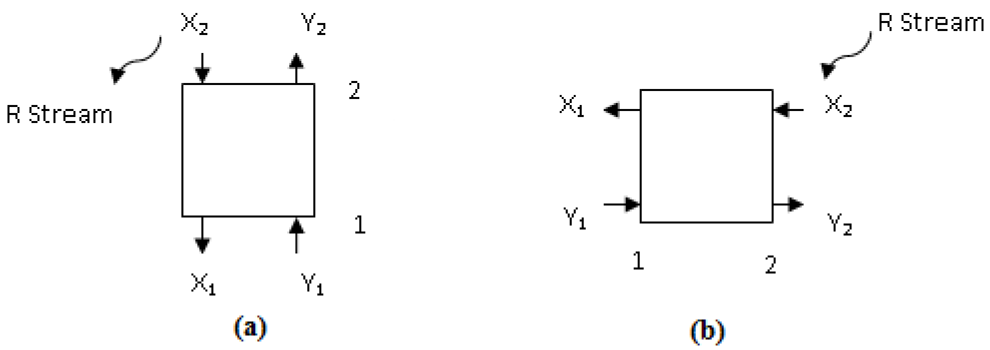

Figure 4 . (a) Vertical gas-liquid contactor where R steam at top; (b) horizontal gas-liquid contactor where R-stream at right end

Therefore the number of plates N_p is

$$N_p = \frac{\log\left(\frac{X_2}{X_1}\left(1 - \frac{1}{S}\right) + \frac{1}{S}\right)}{\log(S)} \tag{89}$$

In terms of Absorption factor

$$N_p = \frac{\log\left(\frac{X_2}{X_1}(1 - A) + A\right)}{\log\left(\frac{1}{A}\right)} \tag{90}$$

$$\frac{X_2 - X_1}{X_2} = \frac{S^{N_p+1} - S}{S^{N_p+1} - 1} = \textit{fraction desorbed} = \frac{\text{input composition} - \text{output composition}}{\text{input composition}} \tag{91}$$

III. Conclusion

Absorption and desorption performance equations for gas-liquid contactor in terms of absorption factor and stripping factor, inlet and outlet mole ratios of the streams to the number of stages have been presented. The physical meaning underlying these performance equations were presented explicitly. The derivations presented in this paper can conveniently be used in terms of mole fraction without modification.

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